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Reynolds Stress Closure in Jet Flows Using Wave Models

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1. Introduction

During this phase of the research program we have continued to develop ways of implementing the turbulence closure scheme based on modeling the large scale coherent structures as instability waves. At the same time we have developed the computational tools necessary to apply this scheme to jets of arbitrary geometry. This has emphasized the use of conformal mapping to transform the irregular physical domain into a convenient computational domain. We have also extended the model we developed earlier for the shock structure of supersonic jets of arbitrary geometry and multiple jets. Further details of the most recent developments are given below.

The greatest difficulty we have encountered so far has been determining the details of how to implement the turbulence closure scheme. We have tried several different approaches. Until recently we have found that though the qualitative features of the unsteady flow field could be predicted there were always difficulties with some of the quantitative features. This has led to a new formulation of the closure scheme. The model still relies on the modeling of the large scale coherent structures as instability waves but does not require a detailed modeling of the small scale fluctuations. We feel that this represents a new and promising feature of our closure technique and we are presently working to implement this scheme.

We have had much more tangible success in the development of the computational tools. This is particularly true for the mapping techniques. The schemes we have developed are very efficient and represent the application of very powerful mathematical tools to problems of practical significance.

We anticipate significant results in the next stage of our research program as the program reaches fruition. Two of the three graduate students involved in this research have passed their comprehensive examinations and are able to devote full time to their research. The third student will take his comprehensive examination this semester.

2. Research Progress

2.1 Turbulence Closure Scheme

Our calculations have shown that it is very important for the present closure model to calculate accurately the amplitude of the wavelike turbulent fluctuations which dominate free shear flows. For free mixing layers, the amplitude is calculated using the kinetic energy equation for the large scale turbulent fluctuations. This equation determines the amount of the wave energy convected at each location by balancing the energy gained from the mean flow with the energy transferred to the small scale, or random fluctuations.

The energy production from the mean shear is obtained explicitly, since we are calculating the Reynolds stresses from the large-scale motions through the Rayleigh equation. However, in our previous approaches we needed to know how to model the residual stresses γ_{ij} in order to calculate the energy transferred from the large to the small scale motions. It was stated in the previous reports that this term is of crucial importance in determining the wave amplitude. We had earlier proposed a simple eddy viscosity model in which the length and the velocity scales were those of the large-scale motions. It was then found that this failed to drain enough energy from the large scale fluctuations. The model was then further improved by using a *split-spectrum* hypothesis. The kinetic energy of the small-scales was obtained by simultaneously solving the mean flow equations and the kinetic energy equation for the small-scales. Using the characteristic velocity thus obtained for the small-scale turbulence in the eddy viscosity model for the residual stresses, the shear layer reached a state of equilibrium. The amplitude of the large scale motions saturated, as we expected them to do, only if we allowed the numerical constant in the eddy viscosity model to grow with the amplitude of the large-scale motions. This suggested that the energy transferred from the large-scale to the small-scale or the high frequency part of the spectrum be proportional to the cube of the velocity scale, i.e. $\frac{u^3}{l}$. Dimensionally, this is in fact the energy dissipated by viscosity at the high wavenumber end of the spectrum.

It thus suggested that we consider an equilibrium state for the small-scale fluctuations in which the rate at which energy is tranferred from the large scales is equal to the rate at which energy is dissipated.

With these observations in mind we have reformulated our approach to the closure scheme. The large scale fluctuations continue to be described by the characteristics of the locally most unstable instability wave. The details of this fluctuation are found from a solution of the Rayleigh equation. The amplitude of the large scale fluctuation is obtained from the solution of the energy integral equation for the wave. In this equation the rate at which energy is lost by the large scales is taken as proportional to the cube of the wave amplitude. The essential difference in the new scheme is that the small scale Reynolds stress is not modeled explicitly. Rather, it is obtained from an integral of the mean momentum equations. It should be noted that once the mean flow and the large scale Reynolds stresses are defined the small scale Reynolds stress is the only unknown in the mean momentum equations. Thus the iterative procedure to be employed as we march downstream from the splitter plate or nozzle consists of inputting the initial mean velocity profile and the amplitude of the large scale structures. The distribution of the large structures is then obtained from the Rayleigh equation along with the corresponding Reynolds stress distribution. This fixes the distribution of the small scale Reynolds stress at the initial location. An implicit algorithm is then used for the streamwise marching. As the axial iterations are performed at each step, the downstream velocity profile, the large and small scale Reynolds stress contributions, and the amplitude of the large scale motions will converge. We are currently checking this formulation in the developed region of the shear layer. This has enabled us to determine an appropriate value for the only empirical constant required by the model which sets the rate of transfer of energy from the large to the small scales. We believe that this approach is quite new and we are hopeful that it will lead to good results.

In order to be able to extend these models to jet flows we have also developed a Rayleigh solver for the instability of compressible axisymmetric jets. This gives the velocity and temperature fluctuations due to the large-scale turbulent structures to be applied in the solution of the mean velocity and temperature fields of axisymmetric jets.

2.2 Computational Domains for Jets of Arbitrary Geometry

Efficient methods establishing the computational domains for jets of arbitrary geometry have been developed and applied. The techniques used involve the generation of the conformal mappings which carry standard computational domains onto the cross sections of a given jet. Two topologically distinct cross sections occur in a jet. The first type of cross section corresponds to the annular shear region surrounding the potential core. The computational domain in this case is the circular annulus. The second type of cross section corresponds to the region in the jet downstream of the potential core. In this situation, the standard computational domain is the unit disc.

The standard regions are chosen to support the numerical solution of the equations modeling jet flows. Conformal mapping techniques are applied because they minimize the number of additional terms introduced by the mapping in the transformed equation governing the stability of jets: the Rayleigh equation. The following two sections will outline the techniques applied to generate conformal maps.

2.2.1 The Wegmann Method - Recently, Wegmann, refs. 1 and 2, proposed a very efficient scheme which solved the boundary correspondence problem associated with mapping the unit disc onto a region with a smooth boundary. The boundary correspondence problem is the determination of the conformal map, F, of the image of the unit circle in the computational domain to the smooth curve bounding the shear region in physical space. Once the boundary correspondence function has been computed, F can be determined on the interior of the unit disc using the Cauchy Integral Theorem.

The central ideas in Wegmann's method will now be outlined. Let $z[s(t)]$ be a parameterization of the curve bounding the shear region of a jet flow cross section. It is assumed that $z[s(t)]$ is a Hölder continuously differentiable function with a nonzero first derivative. The goal of the solution to the boundary correspondence problem is to determine a real periodic function, $\eta(t)$, such that

$$z[s(t) + \eta(t)] \tag{1}$$

are the boundary values at $\exp(it)$ of an analytic function F. Here, F will be the desired conformal map carrying the disc onto the shear region. If $\eta(t)$ is assumed to be a small correction to $s(t)$, then the linearized form of (1) is the boundary of the analytic function of interest, F:

$$z[s(t)] + z'[s(t)]\eta(t) = F(\eta(t)) \tag{2}$$

Since $\eta(t)$ is taken to be small, the curve defined by (2) approximates the shear region boundary curve. Wegmann's method is to recast (2) as a Hilbert-Riemann problem and use techniques from the theory of singular integral equations to obtain a solution. The theory of singular integral equations is introduced in Henrici, ref. 3. This approach is performed iteratively, generating a numerical scheme to solve for η in (2). The iteration amounts to updating $s(t)$ in each step as $\eta(t)$ approaches zero. The iterative scheme is a quadratically convergent Newton-like method which is very efficient in both computer time and storage requirements.

Wegmann has also extended this technique to solve the boundary correspondence problem for the transformation that maps the circular annulus onto a doubly connected region with smooth boundary curves, ref. 4. Therefore, this basic scheme may be applied to both types of shear regions existing in jet flows: the simply and doubly connected shear layer cross sections. Figures 1 and 2 show the geometry of the simply and doubly connected conformal maps.

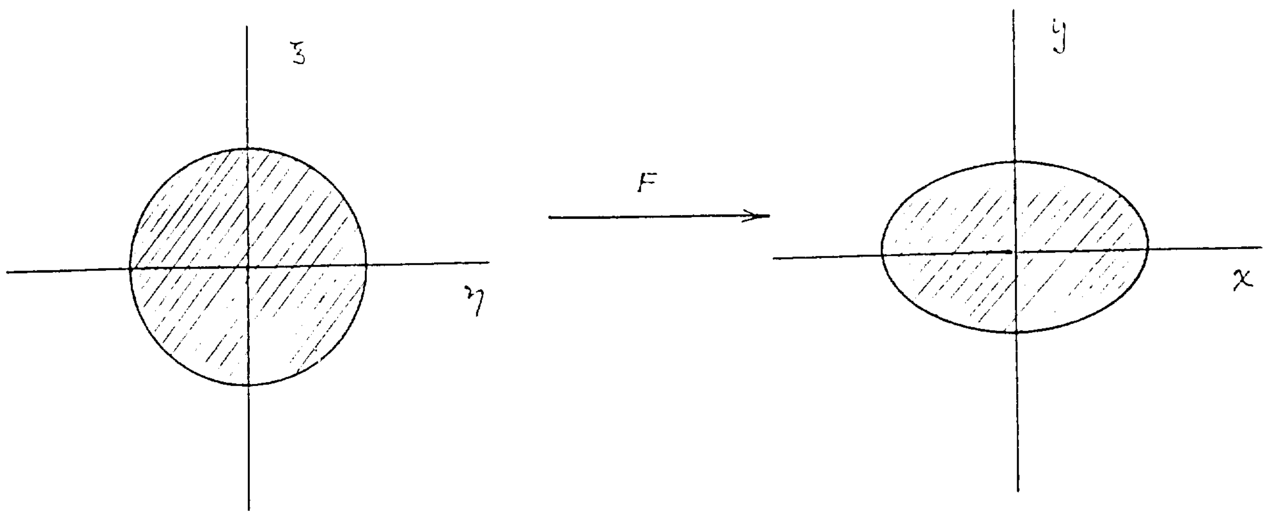

Figure 1: Sketch of Transformation for Simply Connected Domain

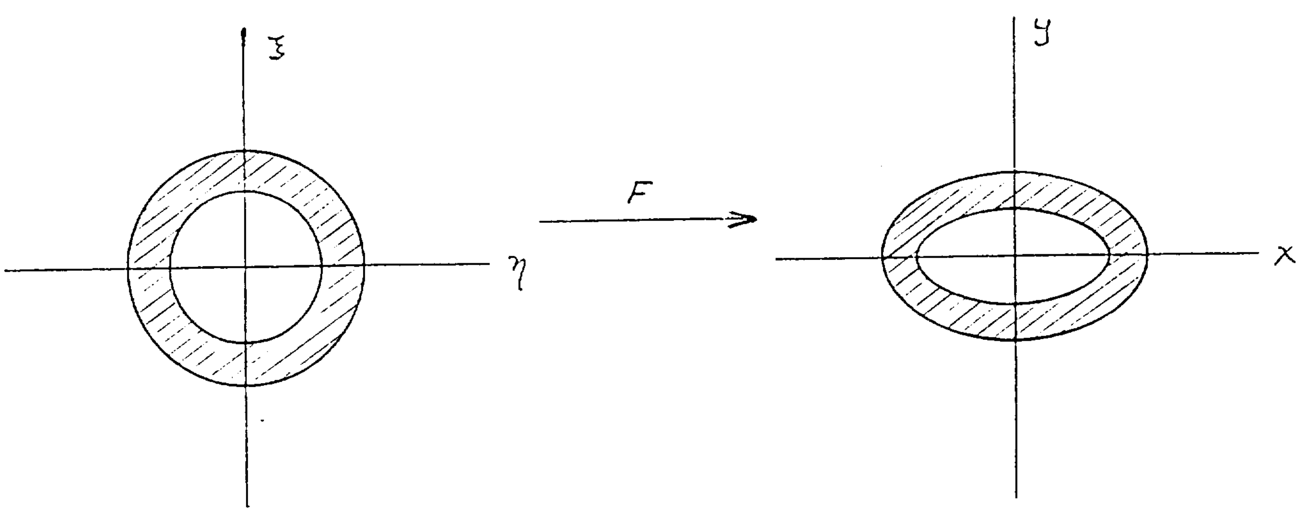

Figure 2: Sketch of Transformation for Doubly Connected Domain

The Wegmann technique has been developed and successfully applied to several examples of elliptic shear regions for aspects ratios up to 4. In particular, Wegmann's method has been applied to the doubly connected shear region close to the end of the potential core. The technique works extremely well for boundary curves which are complex analytic functions. However, this method is very sensitive to the smoothness of the curves bounding the shear region. If a parameterization of the boundary was used which had derivatives which become very large in a small interval, the Wegmann method failed to converge, because $\eta(t)$ was not small. In these cases, Wegmann's method requires very accurate initial guesses for $s(t)$, which are not known in advance. The main example of interest for which the Wegmann method failed was the case of a rectangular shear layer. Here, a simple complex analytic function mapping a standard interval to a smooth approximation of a rectangle is not known.

2.2.2 The Trefethen Method - To construct a conformal transformation from a standard computational domain to a rectangular jet cross section, the Schwartz-Christoffel formula has been applied. In this case, only the portion of the shear layer in the first quadrant of the plane is considered. Then the standard software package, SCPACK, developed by Trefethen, refs. 5 and 6, is used to compute the conformal map from computational space onto a shear layer cross section in physical space. SCPACK efficiently computes the conformal map and its inverse, allowing the determination of the metric tensor. The geometry of the computation for an arbitrary rectangular shear layer is shown in figure 3.

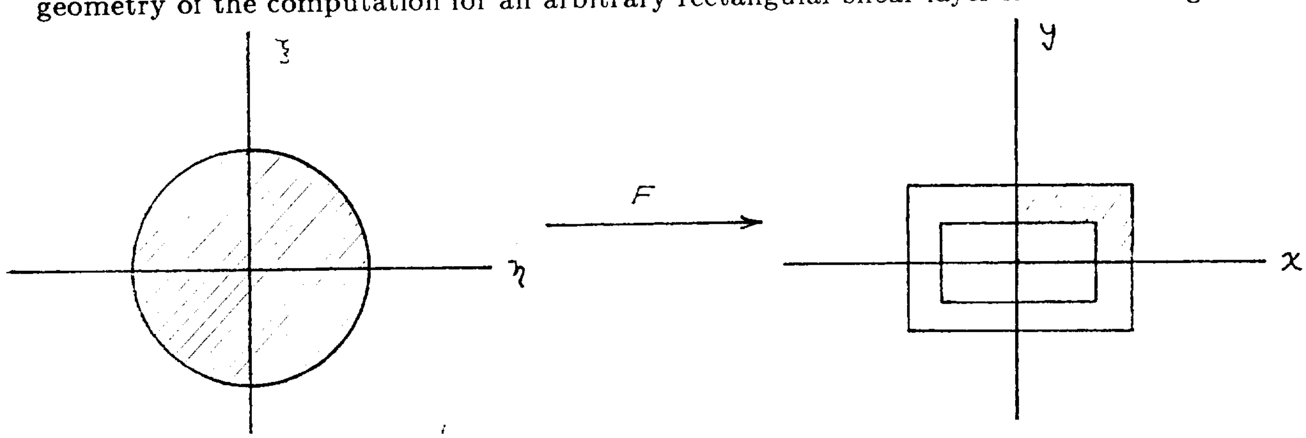

Figure 3. Sketch of Transformation For Rectangular Jet

In the next stage of our calculations the conformal mapping will be combined with the stability calculations, based on the the hybrid spectral method described in previous reports, to determine the local stability characteristics for jets of arbitrary geometry. These calculations will provide the large scale Reynolds stresses needed in our turbulence model.

2.3 Shock Structure in Arbitrary Geometry Jets

Models have been developed to calculate the shock structure and instability waves in jets of arbitrary geometry. The calculation of the shock structure is carried out in the following stages:

(i) A linear shock cell model in which the mixing layer of the jet is approximated by a vortex sheet. This problem may be solved using the boundary element method for general jet geometries.

(ii) The effects of the finite thickness of the mixing layer may be included using a realistic mean velocity and density profile.

(iii) The effects of viscosity and the dissipative effects of the small scale turbulence may be included.

The analysis and the results of the first two stages mentioned above were included in an earlier paper; "Shock Structure in Jets of Arbitrary Exit Geometry" - AIAA Paper 87-2697,1987, (submitted for publication in **J. Sound Vibration**).

At present, the calculations of the shock structure, including the effects of viscosity, for supersonic jets are being carried out. The details of the model developed are presented in the next section.

2.3.1 Development of the model - A finite difference technique has been developed to study the shock structure. This finite difference scheme includes the effects of finite mixing layer thickness through the use of a realistic mean velocity and density profile and the effects of viscosity. We are concerned with the solution of the linearized, compressible equations of motion in which the coefficients depending on the mean flow are arbitrary functions of the plane normal to the jet axis. In the linearized equations the viscous effects or eddy viscous effects describing the influence of the small scale turbulence in the energy equation are neglected and the viscous terms in the momentum equations are characterized by their incompressible form. The mean flow properties are taken to be independent locally of the axial distance: the locally-parallel flow approximation. This enables a separable form of the linearized equations to be obtained.

A body-fitted coordinate system is used which is particularly suited for problems of arbitrary geometry. The coordinate lines are based on the location of the edge of the potential core and its normals. The analytic forms of solution may be found in the potential core region and outside the jet. These are used as the starting conditions for the numerical solution in the mixing layer. The results of the numerical solution are matched with the analytic solutions in the potential core or at the outer edge of the jet. Only certain axial wavelengths will enable the solutions to be matched and these are the eigenvalues for the problem. The initial values for each of the normal modes may be obtained by an eigenfunction expansion at the jet exit, assuming a uniform pressure perturbation. The amplitude of the pressure perturbations in the jet, associated with this initial off-design condition may then be calculated.

At this time the numerical scheme is being validated by comparison with calculations for the circular jet. When this verification is complete the case of the shock structure in the elliptic jet will be considered. The predictions will be compared with experimental observations where possible. The results of these calculations have been submitted for presentation at the AIAA 12th Aeroacoustics Conference, San Antonio, TX in April 1989.

2.4 Multiple Jets

We have extended our analysis of the shock structure and instability of twin supersonic jets to include the effects of a realistic mean velocity profile. This extension enables the shock structure in the jets to be influenced by the adjacent jet. This feature is not possible if the jets are described by vortex sheets. The use of realistic mean velocity profiles also enables the preferred mode of instability of the jets to be determined. It is this frequency component that reaches the greatest amplitude and dominates the near field pressure fluctuations. The interaction between this mode and the steady shock structure, in a phase-locked mode, results in the observed resonant screech phenomenon. The effect of the separation of the jets on the shock structure, the preferred mode and the screech will be calculated. The numerical scheme is being coded and validation tests will then be run. The results of the caculations will also be presented at the AIAA 12th Aeroacoustics Conference.

References

[1] Wegmann, R.: An Iterative Method for Conformal Mapping. **Journal of Computational and Applied Mathematics**, vol. 14, 1986, pp. 7–18.

[2] Wegmann, R.: Convergence Proofs and Errors Estimates for an Iterative Method for Conformal Mapping. **Numerische Mathematik**, vol. 44, 1984, pp. 435-461.

[3] Henrici, P.: **Applied and Computational Complex Analysis**, vol. 3, John Wiley and Sons, 1986.

[4] Wegmann, R.: An Iterative Method for the Conformal Mapping of Doubly Connected Regions. **Journal of Computational and Applied Mathematics**, vol. 14, 1986, pp. 79-98.

[5] Trefethen, L.: Numerical Computation of the Schwarz- Christoffel Transformation. **SIAM Journal of Scientific and Statisical Computation**, vol. 1, 1980, pp. 82-102.

[6] Trefethen, L.: SCPACK User's Guide. ICASE Internal Report, Doc. 24, 1983.

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